A decomposition method to evaluate the 'paradox of progress', with evidence for Argentina

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Abstract

The 'paradox of progress' is an empirical regularity that associates more education with larger income inequality. Two driving and competing factors behind this phenomenon are the convexity of the 'Mincer equation' (that links wages and education) and the heterogeneity in the returns to education, as captured by quantile regressions. We propose a joint least-squares and quantile regression statistical framework to derive a decomposition to evaluate the relative contribution of each explanation. We apply the proposed decomposition strategy to the case of Argentina 1992 to 2015.

Keywords: paradox of progress, quantile regression, inequality, returns to education, Argentina.

JEL: J31, C21, I24, J46, O54.

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1 Introduction

Intuitively, more education should be associated with less inequality, provided that access to education contributes to more equal opportunities. Nevertheless, the empirical evidence suggests otherwise: there is a positive association between the level of education and wage inequality, a phenomenon labeled as the 'paradox of progress' by Bourguignon et al. (2004). Recent evidence on this effect can be found in Beccaria et al. (2015).

There are two alternative hypotheses that may help to rationalize this paradox. First, the 'Mincer equation', that links (log) wages with its determinants, is found to be positive and convex with respect to education, hence higher levels of the latter are associated with higher wage inequality. Tinbergen (1972) and Sattinger (1993) are seminal references that may help explain why this convexity occurs, in a labor market that has a differential rent structure, and education serves as a signal to match workers to better jobs. In turn, this effect depends on the relative scarcity of capital stock across sectors. Thus the overall result is that a more equal distribution of education increases wage inequality due to the convexity of the returns to education (Legovini et al. 2005). Empirical papers that explore this line of research are Bourgignon, Ferreira and Lustig (2005) and Battiston, García Domench and Gasparini (2014), among others. Second, and independently of convexity, the paradox of progress may arise due to individual heterogeneity in returns to schooling. Becker and Chiswick (1966) introduce the idea that human capital depends on individual unobservable characteristics, hence returns to education are heterogeneous. And when positively associated with the conditional distribution of earnings, relatively richer individuals get more from education, hence increased levels of education are associated with a higher mean wage and more inequality. Empirical contributions on this line of research are based on quantile regression (QR), and include Buchinsky (1994, 2001), Martins and Pereira (2004), Staneva et al. (2010), Ariza and Montes-Rojas (2019), among others.

Each model implies a theoretical interpretation on the structure and behavior of the labor market, hence it is important to isolate the relative contribution of each of these two factors behind the increased levels of inequality associated with more education. This paper proposes an econometric framework that encompasses these two models and that leads to a natural way to quantify both the absolute and relative contribution to inequality of convexity and heterogeneity. A functional framework is proposed using mean (i.e. ordinary least-squares, OLS) and QR models to estimate the relative contribution of each hypothesis. The proposed method uses functional derivatives as in Firpo et al. (2009) applied to the variance of the logarithm as the indicator of inequality.

The method is applied for the case of Argentina, which offers a relevant empirical illustration due to the large changes in wage inequality observed in the last 30 years. The results show that at the beginning of the 1990's both unequalizing aspects had the same relevance on the wage distribution. However, convexity of the mean returns became more relevant in 1998 and grows gradually in the following decade and a half, when the effect of heterogeneity became statistically irrelevant, around 2015.

This paper is organized as follows. Section 2 summarizes the main approaches to the paradox of progress. Section 3 shows the econometric methodology. Section 4 applies this method to Argentina 1992-2015. Finally, section 5 concludes.

2 Mincer equation, convexity and heterogeneity

The Mincer equation (Mincer, 1974) is a widely used hedonic price model that postulates that what the market pays for a good depends on its observable, 'hedonic', characteristics. In the labor market, wages depend on time invested to acquire knowledge to apply tasks that require different degrees of complexity. The 'paradox of progress' relates to the empirical finding that higher levels of education are related with higher mean wages and, counter intuitively, with more inequality. As advanced in the Introduction, there are two driving forces behind this phenomenon.

2.1 Convexity of the returns to education

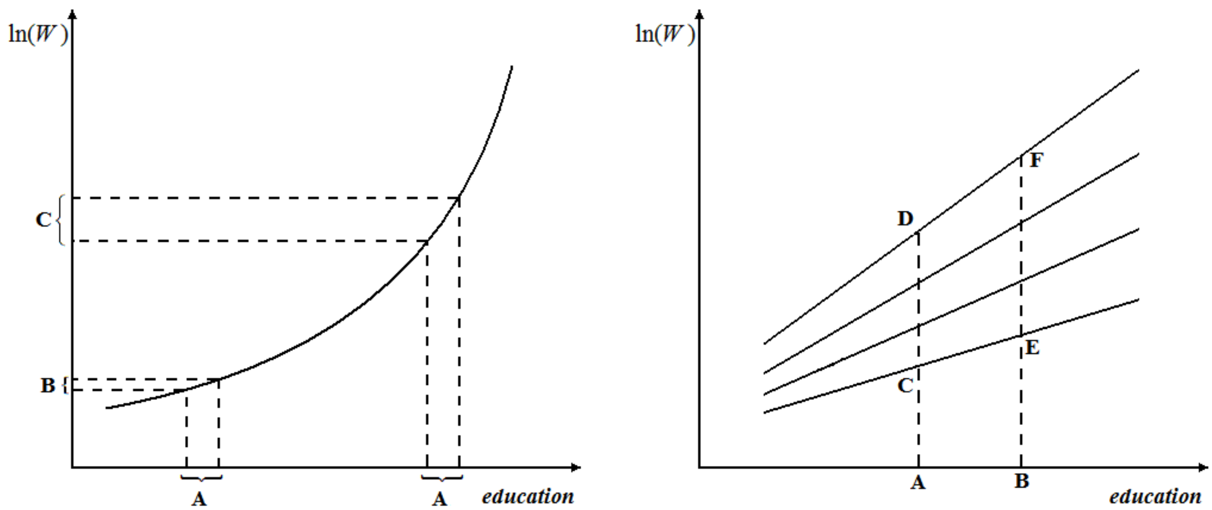
The first hypothesis that rationalizes more education with higher inequality states that, in a partial equilibrium framework, the convexity of the returns to education in the Mincer equation is the cause of the unequalizing effect of increasing education. Figure 1 (a) illustrates this idea. A given increment in education (A) generates a larger effect for higher levels of education (C) than for lower ones (B). The convexity of the Mincer function linking (log) wages with education implies that $C > B$. A formal proof of the result that under convexity more education leads to more unconditional inequality is presented in Appendix A.1.

A seminal theoretical explanation is given by Satinger (1993). The curvature of the wage curve depends on a labor market where there are differential rents and workers self-select into different occupations according to their skills. Years of schooling operate as a signalling mechanism about those skills. Firms have heterogeneous capital stocks and demand those skills. The matching is done by assuming that more capital is associated with higher skills requirements. In equilibrium, there is a positive association between wages and education and the functional form between them depends on the distribution of skills and capital stock. If capital has more dispersion than skills, then there is relative scarcity of high skilled workers. As a result, the market pays them relatively more vis-à-vis low skilled workers. The same model allows for the possibility of an increasing but concave function.

The convexity of Mincer equations with respect to education is a common empirical feature. The procedure to study this effect is by 'microsimulations', that is, by first estimating mean based Mincer equations with household surveys, and then creating a simulated (log) wage arising from assigning one additional year of education to all individuals in the sample. These

Figure 1: Mincer equations - Distributional effects of education

(a) Convexity (b) Heterogeneity



models typically assume homogeneous (and parametric) returns to schooling in a non-linear fashion to allow for convexity. Bourgignon, Ferreira and Lustig (2005) and Battiston, García Domench and Gasparini (2014) are examples of this line of research, whose results provide strong evidence of the convex relation between (log) earnings and education, that may help explain the paradox of progress.

2.2 Heterogeneity in returns to education

An alternative way to rationalize the paradox of progress focuses on the role of unobservable factors interacting with education in the determination of wages. Becker and Chiswick (1966) discuss a model where each individual has her own human capital, hence returns to education are individual specific. When education interact with these unobserved factors, returns to education might increase with them. For example, individuals with better (unobserved) family background may benefit more from education than those in worse conditions.

The mean based models usually adopted in the 'microsimulation' approach previously discussed do not capture these distributional effects of an increment in education. Instead, *quantile regression* models have been have been utilized to accomodate heterogeneous returns. Consider the example of Figure 1 (b). There are four different wage curves, one for each of four specific quantiles of the conditional distribution of wages. The fact that the slopes of these curves are increasing illustrates the idea that individuals up in the conditional distribution (for example, with better family background) face higher returns to education. The wage gap for individuals with education in A is $D - C$, smaller than that of those with education B, $F - E$. Hence, more education leads to more conditional 'within' inequality, that eventually translates into more unconditional wage dispersion. A formal proof that in this context more education leads to higher unconditional inequality under heterogeneity is shown in Appendix A.1.

An extensive line of research including Buchinsky (1994, 2001), Martins and Pereira (2004), Staneva et al. (2010), among many others, provide strong evidence of this empirical pattern where wage returns are increasing along conditional quantiles, hence the wage gap is increasing in education. Different econometric analyses rely on micro simulations to explore increases in unconditional inequality from its conditional counterpart arising from standard quantile regression, as in Autor, Katz and Kearney (2005), Machado and Mata (2005), Melly (2005), Montes-Rojas, Siga and Mainali (2017), among others. A more recent alternative approach is to handle unconditional effects directly, as in the recentered influence function (RIF) regression method of Firpo, Fortin and Lemieux (2009,2018), that leads to 'unconditional' quantile regressions.

In any case, in their current state, mean models focus on measuring the contribution of convexity, and their QR counterpart on that of heterogeneity. In the next section we propose a modeling strategy that encompasses both ideas and leads to a natural way to quantify the relative importance of each of the factors behind the paradox of progress.

3 A decomposition approach for the paradox of progress

In this section we present a methodology to decompose the two potential effects driving the paradox of progress. It is based on a simple re-parameterization of a QR model using cross-sectional data, based on Autor, Katz and Kearney's (2005) model. We further assume the exogeneity of the covariates (and in particular schooling). The proposed method can be used with instrumental variables if available to tackle endogeneity as well. Following the empirical literature we consider only partial equilibrium effects, which means that changes in the composition of education does not affect factor prices. In this sense, our approach is similar to that adopted by Firpo et al. (2009) where the conditional distribution can be used together with changes in covariates to study the unconditional wage distribution.

3.1 Population model

Assume that wages are represented by the following random coefficient representation of QR (Koenker and Xiao (2006), Montes-Rojas et al. (2017))

$$W = X'\alpha(U), \tag{1}$$

where W is the logarithm of wages, X is a set of observable attributes (education, experience, gender, etc.), $\alpha()$ is a strictly monotonic function and $U|X$ is a random variable with standard uniform distribution representing the effect of unobservable components[1]

The 'index' U represents the ranking of individuals along the distribution of wages conditional on X. This ranking is determined by the unobservable components that explain wage heterogeneity (ability, luck, family background, etc.). Therefore, the functional form of $\alpha(.)$ depends on the joint distribution of X and U. In turn, the vector or random coefficients $\alpha(.)$ can be interpreted as modeling the interaction of observables and unobservables.

Following Autor, Katz and Kearney (2005), let $E(W|X = x) = x'\beta$ be the conditional mean model. Then we can write (1) as

$$W = X'\beta + X'\gamma(U), \tag{2}$$

defining $\gamma(U) := \alpha(U) - \beta$. In other words, $\gamma(U)$ is the difference between the quantile-specific and the mean effects. In this setup, discrepancies in W are due to *between* differences associated to different levels of X, and to *within* differences, that is, wage disparities associated to different wages for the same level of X. We will use this definition of the between and within effects in the rest of the paper.

This model will be rewritten in a form that contains the two frameworks outlined before for explaining the paradox of progress. First, following Buchinsky (1994) the heterogeneity in wages can be analyzed in terms of how U affects wages through α or γ. Second, following Bourgignon, Ferreira and Lustig (2005) heterogeneity in wages could arise due to a non-linear (convex) functional form in X. For this we consider a simple model with $X = [H, H^2, Z]$, where H is a continuous measure of human capital (i.e. years of schooling) and Z other covariates.

The relevant measure of wage inequality (I) will be the variance of logs. In general, this measure correlates positively with other measures of inequality (e.g. Gini, Theil), see Cowell (2000). Using the law of total variance (see Angrist and Pischke (2009, p.33)):

$$I := Var(W) = Var[E(W|X)] + E[Var(W|X)]. \tag{3}$$

Then, calculating $E(W|X)$ and $Var(W|X)$ for our model (see Appendix A.2 for details) and replacing we obtain the following wage decomposition:

$$I = \beta' V \beta + tr(\Omega V) + E'\Omega E, \tag{4}$$

where V is the variance-covariance matrix of X, E is the vector of (unconditional) means of X, and $\Omega = Var[\gamma(U)]$ is a measure of the distance of the mean and quantile coefficients.

[1]See the proof in Appendix 2.1 for details on the random coefficients representation of quantile regressions.

3.2 Decomposing the marginal effect of education

Given that (4) contains both the mean (E) and the variance (V) of X, the measure of unconditional inequality I not only depends on parameter heterogeneity but also on how covariates are distributed. Following Fortin et al. (2009, 2018) we will consider a small translation (location shift) in H, years of schooling, to $H + \epsilon$ with $\epsilon \to 0$. Let $\delta(I(X)) := \lim_{\varepsilon \to 0} \frac{I[(X+\varepsilon)]-I(X)}{\varepsilon}$ denote the marginal effect of this location shift on inequality, I.

Assuming that the parameters β and γ are not affected by this shift, then the total effect of a location shift in X on inequality can be expressed as follows (see Appendix A.3 for a derivation):

$$\delta(I) = \beta'\delta(V)\beta + tr[\Omega\delta(V)] + 2E'\Omega\delta(E). \tag{5}$$

Consequently, changes can be decomposed into a *between* (convexity) and a *within* (heterogeneity) effect as follows:

$$EF_{betw} = \beta'\delta(V)\beta,$$
$$EF_{with} = tr[\Omega\delta(V)] + 2E'\Omega\delta(E),$$

such that $\delta(I) = EF_{betw} + EF_{with}$. In this setup education alters inequality through the average wage gap between different groups according to the levels of X, and from the within group, for the same X, due to differences across quantiles of the conditional distribution (log) wages.

In order to fix ideas, consider the following simple example. For simplicity assume that $X = [H, H^2]$. Then,

$$W = \beta_0 + \beta_1 H + \beta_2 H^2 + \gamma_0(U) + \gamma_1(U)H + \gamma_2(U)H^2.$$

Applying the decomposition discussed above:

$$EF_{betw} = 4(\beta_1 V_{11} + \beta_2 V_{12})\beta_2,$$
$$EF_{with} = 2[\Omega_{01} + 2\Omega_{02}E_1 + 3\Omega_{12}E_2 + \Omega_{11}E_1 + 2\Omega_{22}E_3],$$

where V_{ij}, Ω_{ij} and E_i refer to the corresponding $i, j = 0, 1, 2$ or $i = 0, 1, 2$ elements of V, Ω, or E. Convexity is led by $\beta_2 \neq 0$. Note that even though convexity is the leading factor behind the between inequality it does not necessarily play a role in the within effect. Hence, education might increase inequality through the within channel even when the effect of education is linear in the 'mean' part of the model ($\beta_2 = 0$).

Consider the following three illustrative cases:

- **Case 1 (linear homoskedastic model)**: $\beta_2 = 0$ and Ω has zeros except for $\Omega_{00} > 0$. Then,

$$EF_{betw} = EF_{with} = 0.$$

- **Case 2 (quadratic homoskedastic model)**: $\beta_2 > 0$ and Ω has zeros except for $\Omega_{00} > 0$. Then,

$$
\begin{aligned}
EF_{betw} &= 4V_{12}\beta_2^2 > 0,\\
EF_{with} &= 0.
\end{aligned}
$$

- **Case 3 (linear heteroskedastic model)**: $\beta_2 = 0$ and the diagonal elements of Ω are non zero.[2] Then,

$$
\begin{aligned}
EF_{betw} &= 0,\\
EF_{with} &= 2\Omega_{11}E_1 + 4\Omega_{22}E_3 > 0.
\end{aligned}
$$

3.3 Estimation and inference

The proposed decomposition leads to a simple way to quantify and separate the relative contribution of the factors behind the paradox of progress. This subsection describes how to implement it in practice with a sample of wages and its determinants. Let $\{w_i, x_i\}_{i=1}^n$ be a random sample of waged employees, where x contains h (education) and h^2 and other individual characteristics z. The parameters β and $\alpha(\tau)$, $\tau \in (0,1)$, are estimated by OLS and QR as in Koenker and Basset (1978), respectively.

In order to estimate Ω, the variance-covariance matrix of $\gamma(U) = \alpha(U) - \beta$ where U is uniformly distributed on $[0,1]$, we use the following procedure. Consider a grid of M indexes, $0 < \tau_1 < \tau_2 < ... < \tau_M < 1$, and let $\alpha(\tau_m)$ be the corresponding vector QR coefficients. Then,

$$
\hat{\Omega} = M^{-1} \sum_{m=1}^{M} [\hat{\alpha}(\tau_m) - \hat{\beta}] \cdot [\hat{\alpha}(\tau_m) - \hat{\beta}]'.
$$

If M is large enough, which in turn should depend on n, i.e. $M = M_n$, then $\hat{\Omega} \xrightarrow{p} \Omega$ as $n \to \infty$. We propose a similar procedure to Chernozhukov, Fernández-Val and Melly (2013), where they impose that $t = \tau_1$ and $\tau_M = 1 - t$ for a small $t > 0$ (to avoid estimating extreme quantiles) and the quantiles are estimated on a fine mesh with width $\psi = \tau_j - \tau_{j-1},\ j = 2, ..., M$, with $\psi\sqrt{n} \to 0$ as $n \to \infty$.

Finally, consider estimators of E and V. Let Q be the number of regressors included in Z, then we obtain $\delta(E)$ and $\delta(V)$ (see Appendix A.4 for a derivation):

[2] We could also consider the non-diagonal elements for a more involved model.

$$\delta(E) = \begin{bmatrix} 0 \\ 1 \\ 2E_1 \\ 0_{1\times Q} \end{bmatrix} \qquad \text{and} \qquad \delta(V) = \begin{bmatrix} 0 & 0 & 0 & 0_{1\times Q} \\ 0 & 0 & V_{11} & 0_{1\times Q} \\ 0 & V_{11} & 2V_{12} & M_{1z} \\ 0_{Q\times 1} & 0_{Q\times 1} & M_{z1} & 0_{Q\times Q} \end{bmatrix} \tag{6}$$

where $E_1 = E(h)$, $V_{12} = Cov(h, h^2)$, $M'_{z1} = 1z = Cov(h, z)$ ($1 \times Q$ vector) and $0_{A\times B}$ is a null matrix of dimensions $A \times B$. All these components can be consistently estimated as:

$$\hat{E} = n^{-1} \sum_{i=1}^{n} x_i$$
$$\hat{V} = (n-1)^{-1} \sum_{i=1}^{n} (x_i - \hat{E}) \cdot (x_i - \hat{E})'.$$

Then, replacing in (5),

$$\hat{\delta}(I) = \hat{\beta}'\delta(\hat{V})\hat{\beta} + tr[\hat{\Omega}\delta(\hat{V})] + 2\hat{E}'\hat{\Omega}\delta(\hat{E}). \tag{7}$$

The first term in (7) is the estimation of the convexity part of the paradox of progress and the last two estimate the heterogeneity effect.

Bera, Galvao and Wang (2014) study the asymptotic properties of the joint estimators of OLS and QR estimators, and derive uniform weak convergence of the joint process indexed by the quantile index. They also study the conditions for using wild bootstrap methods for parameter inference. Montes-Rojas, Siga and Mainali (2017) use this strategy to study parameter heterogeneity comparing mean and quantile coefficients. For the purposes of this paper, the proposed measure of inequality and its components are continuous transformations of estimators of OLS and QR coefficients, hence the continuous mapping theorem applied to the components of (7) guarantee the existence of a stable asymptotic distribution and justifies the use of the bootstrap procedures. In particular, we apply wild bootstrap methods by resampling the original sample by individual without replacement.

4 Exploring the paradox of progress for the case of Argentina

In this section we implement the proposed decomposition methodology with data from the Permanent Household Survey (EPH, acronym in Spanish) implemented by the National Institute of Statistics and Censuses (INDEC) of Argentina. The decomposition is computed in four distant years to explore different moments of the Argentine wage distribution and to evaluate long-run changes: 1992, 1998, 2008 and 2015. The criterion for choosing these years is due to data availability and to consider periods with a relative macroeconomic stability and similar survey and sampling methodologies. The first two years belong to the so-called discontinuous survey methodology, usually carried out in the months of May and October, while the last two years correspond to the continuous survey methodology, carried out quarterly. INDEC has been expanding the sampling coverage over the period of analysis, and therefore we only use the observations that belong to the urban agglomerates present in the 1992 EPH to keep all estimates comparable. The data correspond to the surveys collected during the second semester with the exception of 2015, which is when the EPH changed its survey methodology in the second half of the year and therefore we used the first semester for comparability reasons. The sample used in all cases is of men between 16 and 65 years of age.

Table 1 presents the estimated coefficients for the education variables of the Mincer equations of the conditional mean and some relevant conditional quantiles. These regressions are only descriptive, for the decomposition a much larger number of quantiles is used. Other typical covariates from the Mincer equations literature are also included: potential experience (and its square), marital status, and controls by geographic region. Quadratic terms are statistically significant in the four years considered and at the different points of the conditional distribution. This means that the relationship between wages and educational level is convex.

Additionally, the coefficients of the conditional quantiles are not constant, suggesting a heterogeneous pattern in the returns to education, similar to that postulated in Becker and Chiswick (1966). For example, comparing the coefficient of the quadratic term between conditional deciles, the difference is high in 1992 and 2015 (the last decile is at least 2.5 times the first), slight in 1998 (1.3 times) and null in 2008. Figure 2 shows this result graphically comparing the predictions of each Mincer regression equation (keeping the rest of the covariates in their sample means). Convexity appears to be more relevant in 1998, while the pattern of heterogeneity in returns to education has been disappearing over the last fifteen years.

The presence of heterogeneity and a convex relationship between (log) wages and education indicates that increased education is associated with higher unconditional inequality, as discussed in Section 2. In order to quantify the strength of these effect, and to decompose their relative importance in inequality, Table 2 presents the results of the decomposition methodology introduced in this paper. The first two rows of the table shows the evolution of wage inequality for men aged 16 to 65, measured by the Gini index and the variance of the logarithms. Both indicators show a similar evolution: an increase in wage inequality towards the end of the 1990s, a relevant distributional improvement in the 2000s, slightly sustained towards the middle of the last decade.

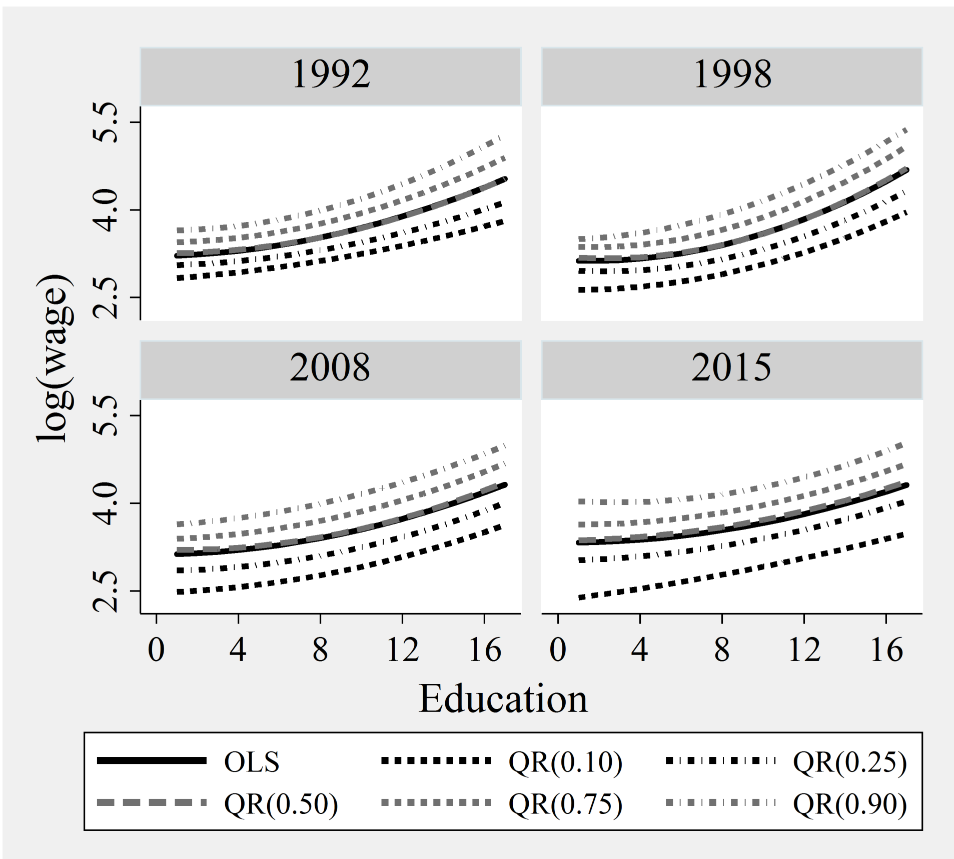
The second block of Table 2 presents a microsimulation exercise similar to those in the literature based on the conditional mean. It uses the Mincer equation estimated by OLS and its residuals to re-compute a new wage distribution after a small horizontal translation of education is implemented. Specifically, if an individual i has an education level of h_i, the exercise is to impute $h_i + \varepsilon$ years of education and construct a counterfactual salary (in

Table 1: Partial relationship between wage (log) and educational level. Argentina 1992 - 2015.

	OLS	**QR(0.10)**	**QR(0.25)**	**QR(0.50)**	**QR(0.75)**	**QR(0.90)**
				1992 ($n = 12196$)		
Education	0.0070	0.0228***	0.0062***	-0.0022***	0.0006	-0.0049***
	(1.00)	(26.08)	(8.89)	(-5.05)	(0.80)	(-4.95)
Education^2	0.0042***	0.0021***	0.0034***	0.0045***	0.0050***	0.0059***
	(12.73)	(52.03)	(103.12)	(219.24)	(152.62)	(127.97)
				1998 ($n = 11228$)		
Education	-0.0198***	-0.0056***	-0.0289***	-0.0334***	-0.0240***	0.0035***
	(-2.76)	(-4.94)	(-52.82)	(-91.66)	(-70.88)	(6.04)
Education^2	0.0065***	0.0049***	0.0063***	0.0072***	0.0073***	0.0063***
	(19.58)	(93.00)	(249.73)	(423.64)	(464.61)	(237.31)
				2008 ($n = 14580$)		
Education	0.0034	0.0103***	0.0004	-0.0103***	0.0071***	0.0168***
	(0.51)	(14.10)	(0.83)	(-28.85)	(15.52)	(25.14)
Education^2	0.0039***	0.0034***	0.0040***	0.0046***	0.0041***	0.0037***
	(13.12)	(104.41)	(209.74)	(286.76)	(198.73)	(125.35)
				2015 ($n = 14553$)		
Education	-0.0010	0.0432***	0.0063***	0.0018***	-0.0065***	-0.0279***
	(-0.14)	(41.27)	(10.41)	(4.53)	(-12.42)	(-57.34)
Education^2	0.0035***	0.0014***	0.0032***	0.0034***	0.0040***	0.0050***
	(11.28)	(29.98)	(118.39)	(192.49)	(171.48)	(233.17)

Notes: Other variables included are: potential experience (and its square), marital status, and controls by geographic region. The t statistics are shown in parentheses, * indicates significance at 10%, ** at 5% and *** at 1%.

Figure 2: Convexity and Heterogeneity of the returns to education.



Source: own estimates based on the EPH.
Note: the other covariates are evaluated at their sample means.

Table 2: Marginal effect of education on inequality.

	1992	**1998**	**2008**	**2015**
1. Inequality				
Gini index	40.5	44.0	39.8	35.6
Variance of logarithms	42.5	53.8	48.5	42.1
2. Microsimulation (location shift)	1.82**	3.69**	1.71**	1.17**
	(0.33)	(0.39)	(0.24)	(0.19)
3. RIF estimate	4.65**	6.36**	2.03**	1.36**
	(0.51)	(0.56)	(0.39)	(0.38)
4. Decomposition (levels)				
Between (convexity)	1.81**	3.69**	1.71**	1.17**
	(0.33)	(0.39)	(0.23)	(0.19)
Within (heterogeneity)	1.88**	1.47**	0.56*	0.540
	(0.28)	(0.32)	(0.28)	(0.30)
Total change	3.69**	5.16**	2.27**	1.71**
	(0.48)	(0.55)	(0.35)	(0.34)
5. Decomposition (percentage)				
Between effect (convexity)	49.1%	71.5%	75.3%	68.4%
Within effect (heterogeneity)	50.9%	28.5%	24.7%	31.6%
Total change	100%	100%	100%	100%

Notes: The t statistics are shown in parentheses, * indicates significance at 10%, ** at 5% and *** at 1%.

logarithms) w_i^s through the prediction of the conditional mean and adding the OLS residual. Then, the marginal change in inequality is calculated as $[V(w^s) - V(w)]/\varepsilon$, where ε is a small value.[3] The next block of Table 2 contains the change in variance estimated with the Firpo et al. (2009) methodology through RIF regression for the variance.[4] Both methodologies show an unequalizing effect of education on wages, reaching its maximum impact in the late 1990s. Although both strategies measure the effects of a small location shift on education, the gap between both is that the RIF regression is more accurate because it contemplates the entire information of the conditional distribution of wages, while the numerical simulation only extrapolates the behavior of the conditional mean. Nevertheless, neither methodology lead to a natural way to separate the contributions of convexity and heterogeneity towards increased inequality.

The fourth and fifth blocks of Table 2 show the results of implementing our proposed decomposition. We set a grid of values for $\tau = 0.005, 0.01, \ldots, 0.99, 0.995$ for the QR estimation, involving a total of $M = 199$ estimated equations for each year.[5] The results show that the total effect is quite similar to the estimate made by the RIF method. The contribution of our decomposition is that it explicitly shows the relative weight of the empirical explanations of the unequalizing effect of education based on both the conditional mean and the conditional quantiles.

Almost all terms in the decomposition terms are statistically significant at the usual levels. At the beginning of the 90s convexity and heterogeneity had the same relevance on the increased inequality due to more education. However, the effect of convexity on the mean returns becomes more relevant in 1998 (just over 70%) and seems to grow gradually in the following decade and a half, where the effect of heterogeneity becomes statistically irrelevant in 2015.

5 Concluding remarks

This paper proposes a decomposition methodology for the marginal effect of education on unconditional wage inequality. The method quantifies the relevance of two empirical arguments previously outlined in the literature to explain the unequalizing effect of education, labeled as the 'paradox of progress'. The first one is based on the convexity of the Mincer equation while the other one focuses on the heterogeneity of returns to education at different levels of the conditional distribution of (log) wages.

Our decomposition is based on a 'functional derivative' as in in Firpo et al. (2009) applied

[3]The value used in this paper is $\varepsilon = 0.01$.

[4]The recentered influence function (RIF) regression method estimates the effect of covariates on different functionals of an unconditional distribution. In this case, we consider the effect of the same set of covariates on the unconditional variance of log wages. Although not reported, similar qualitative results are obtained if using the RIF regression for Gini.

[5]This number of quantiles is similar to Melly (2005) for generating counterfactual distributions with many QR models.

to the variance of the logarithms as the indicator of inequality. Our proposal only requires consistent estimates of the parameters of the conditional mean and quantiles. The method applies naturally to any other consistent regression mehtod (instrumental variables, panel data, etc.).

The implementation of the decomposition for the case of Argentine shows that in the early 1990s both convexity and heterogeneity mattered alike as unequalizing factors due to increased education. Still, towards the end of the studied period, the effect of convexity dominates. This change in the relevance of the two unequalizing forces could give rise to some interpretations of how the functioning of the labor market has changed. On the one hand, the increase in the relevance of convexity in the returns to education could be indicating a certain relative scarcity of supply in the stock of available human capital in relation to the variety of the type of qualified tasks demanded by employers. Consequently, the market pays more than proportionally more educated workers, who are likely to be able to work on multiple tasks. On the other hand, the decreasing role of the conditional heterogeneity in returns to education would be associated with a less important role of the market's unobservable wage determinants (innate skills, tenacity, intelligence, luck, etc.). This would indicate some direct loss in the return received by these unobservable factors in the market, or indirectly due to a lesser complementarity of these with the educational level of the individuals. A detailed study of these mechanisms is a relevant topic of further research.

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Appendix

A.1 Convexity and heterogeneity lead to higher unconditional inequality

In this Appendix we establish two results. The first one shows that a location shift in a convex Mincer equation leads to more unconditional inequality. The second one proves that a location shift in a linear quantile regression model with increasing heterogeneity leads to more unconditional inequality. We make use of two results in See and Chen (2008).

Convexity leads to higher inequality: Let $Y = f(X + \epsilon)$, where X is a random variable and f is a differentiable, increasing and convex function. Then

$$\frac{d\,V(Y)}{d\,\epsilon} \geq 0$$

Proof: Start from

$$V[f(X + \epsilon)] = E\{f^2(X + \epsilon)\} - E^2[f(X + \epsilon)].$$

Taking derivatives with respect to ϵ, and using Lemma 2.2 in See and Chen (2008),

$$\tfrac{d}{d\epsilon}V[f(X + \epsilon)] = E\{2f(X + \epsilon)\tfrac{d}{d\epsilon}f(X + \epsilon)\} - 2E[f(X + \epsilon)]E[\tfrac{d}{d\epsilon}f(X + \epsilon)].$$

Let $h(X) := f(X + \epsilon)$ and $g(X) := \frac{d}{d\epsilon}f(X + \epsilon)$, then

$$\tfrac{d}{d\epsilon}V[f(X + \epsilon)] = 2E\{h(X)g(X)\} - 2E[h(X)]E[g(X)] \geq 0,$$

by Lemma 2.1 in See and Chen (2008), since both h and g are increasing under the assumptions about f.

Heterogeneity leads to more inequality: Now assume $Y = f(X + \epsilon, U)$, where U represents unobserved heterogeneity and X is a scalar random variable. Consider the following linear quantile regression model $f(X + \epsilon, U) = a(X + \epsilon) + b(X + \epsilon)\ g(U)$, with $a(X + \epsilon) = a_0 + a_1(X + \epsilon)$, where a_0 and a_1 are scalars, b and g are positive and increasing functions, and $U|X \sim \text{Uniform}(0, 1)$ and independent of X. This is the representation of linear quantile regressions proposed in Koenker and Xiao (2006), as non-linear functions of uniform random variables. More concretely, if, in general $Y = X'\alpha(U)$, and $\alpha(.)$ is an increasing monotonic function, then $Q_{Y|X}(\tau) = X'\alpha(Q_{Y|X}(\tau))$ -the standard QR representation-, since quantiles are equivariant under monotonic transformations and $U|X$ is uniform.

Assume $E[g(U)] = 0$ and $V[g(U)] = \sigma_g^2$. Then

$$\frac{d\,V(Y)}{d\,\epsilon} \geq 0.$$

Proof: Start from the 'law of total variance':

$$V(Y) = V[E(Y|X)] + E[V(Y|X)].$$

Note that $E(Y|X) = a_0 + a_1\ (X + \epsilon)$ and $V(Y|X) = b(X+\epsilon)^2\sigma_g^2$. Then,

$$V(Y) = a_1^2\ V(X) + E[b^2(X+\epsilon)]\sigma_g^2.$$

Using Lemma 2.2 in See and Chen (2008),

$$\tfrac{d}{d\epsilon}V(Y) = E[2b(X)c(X)]\sigma_g^2 = 2E[b(X)c(X)]\sigma_g^2,$$

where $c(X) := \frac{\partial}{\partial\epsilon}b(X+\epsilon)$. The result follows since b is positive and increasing.

A.2 Derivation of eq. (4)

Consider equation (1) and calculate the expectation of W conditional on $X = x$, then

$$E(W|x) = x'E[\alpha(U)|x] = x'E[\alpha(U)] := x'\beta. \tag{a.1}$$

Note that β has been defined as the expectation of the random vector $\alpha(U)$, where $U|x$ is $Uniform(0,1)$. That is, the parameters β of the conditional expectation are the average of the parameters of all the conditional quantiles.

On the other hand, consider (2) and compute the conditional variance

$$Var(W|x) = Var[x'\beta|x] + Var[x'\gamma(U)|x] = x'Var[\gamma(U)]x := x'\Omega x, \tag{a.2}$$

where Ω has been defined as the matrix of variances of the vector $\gamma(U)$. Note that by construction the expectation $E[\gamma(U)] = E[\alpha(U) - \beta] = 0$ and therefore $Var[\gamma(U)] = E[\gamma(U)\gamma(U)'] = \Omega$. That is, the matrix Ω is a notion of distance between the mean and the quantiles of the distribution $W|X$.

Combining and using the Law of Iterated Variances,

$$Var(W) = Var[X'\beta] + E[X'\Omega X]$$

Then, using standard properties of variance of the product of vectors and properties of the expectation for quadratic forms:

$$Var(W) = \beta' Var(X)\beta + tr[\Omega Var(X)] + E(X)'\Omega E(X).$$

A.3 Derivation of eq. (5)

(5) is a particular case of the notion of (partial) functional derivative proposed by Firpo et al. (2009). In this literature it is usual to assume that the distribution of $w|x$ is not affected by

changes in the distribution of x. This assumption translated into our quantile model means that the parameters β and Ω do not change as a consequence of a location shift in any of the regressors included in x. Intuitively, this assumption makes explicit the fact that it is a partial equilibrium analysis, in the sense that a small change in education (measured by h) does not change the returns to education. The functional derivative of the inequality I with respect to a horizontal translation in h is obtained by computing a differential limit of equation (3.4). For example, to derive the expression $\beta' V \beta$ (first term of I) we solve the following limit:

$$\begin{aligned}
\delta[\beta' Var(x)\beta] &= \lim_{\varepsilon \to 0} \frac{[\beta' Var(x+\varepsilon)\beta] - [\beta' Var(x)\beta]}{\varepsilon} \\
&= \lim_{\varepsilon \to 0} \frac{\beta'[Var(x+\varepsilon) - Var(x)]\beta}{\varepsilon} \\
&= \beta' \left[\lim_{\varepsilon \to 0} \frac{Var(x+\varepsilon) - Var(x)}{\varepsilon} \right] \beta := \beta' \delta(V) \beta
\end{aligned}$$

Using the previous reasoning but applied to the rest of the terms of equation (4), it follows that:

$$\delta[tr(\Omega V)] = tr[\Omega \delta(V)] \qquad \text{and} \qquad \delta(E'\Omega E) = 2E'\Omega\delta(E)$$

Finally, adding these three components gives equation (5) as a result.

A.4 Derivation of eq. (6)

To obtain the expressions (6) it is convenient to analyze each of the elements in E and V. The matrix V contains all the variances and covariances of the variables included in the vector x, while E is a vector that contains the expectation x:

$$E := \begin{bmatrix} E_0 \\ E_1 \\ E_2 \\ E_z \end{bmatrix} \qquad \text{and} \qquad V := \begin{bmatrix} V_{00} & V_{01} & V_{02} & V_{0z} \\ V_{10} & V_{11} & V_{12} & V_{1z} \\ V_{20} & V_{21} & V_{22} & V_{2z} \\ V_{z0} & V_{z1} & V_{z2} & V_{zz} \end{bmatrix},$$

where the element notation includes the following scalars,

$$E_k = E(h^k) \qquad \text{and} \qquad V_{jk} = Cov(h^j, h^k)$$

for $k = 0, 1, 2$ y $j = 0, 1, 2$, together with the following $(Q \times 1)$ vectors

$$E_z = E(z) \qquad \text{and} \qquad M_{kz} = Cov(h^k, z) = M'_{zk}$$

for $k = 0, 1, 2$, and the $(Q \times Q)$ matrix:

$$M_{zz} = V(z)$$

Note that when $k = 0$ the vector M_{0z} is a null vector, because it is the covariance between $h^0 = 1$ with each of the regressors z.

The terms $\delta(E)$ and $\delta(V)$ are the functional derivatives of each of the elements of E and V, respectively. Consider a location shift ε that only affects the distribution of years of education h. Then we can calculate the functional derivatives of each element in E and V as follows:

(i) *First-order moments of x*

First, analyze the effect on E_k:

$$\delta(E_k) = \lim_{\varepsilon \to 0} \frac{E[(h+\varepsilon)^k] - E(h^k)}{\varepsilon} = E\left[\lim_{\varepsilon \to 0} \frac{(h\varepsilon)^k - h^k}{\varepsilon}\right] = E(kh^{k-1}) = kE_{k-1} \quad \text{(a.3)}$$

In addition, the vector $E_z = E(z)$ does not change with a horizontal translation in ε, therefore it is evident that $\delta(E_z) = 0$. Substituting all this in $\delta(E)$ gives the first part of (3.6).

(ii) *Variance and covariance between h^j and h^k*

The origin of this block of matrix V is the inclusion of h and h^2 as part of the covariates x. Note that $V_{jk} = E_{j+k} - E_j E_k$, then using the result (a.3) follows that:

$$\begin{aligned}
\delta(V_{jk}) &= \delta(E_{j+k} - E_j E_k) \\
&= \delta(E_{j+k}) - \delta(E_k)E_j - \delta(E_j)E_k \\
&= (j+k)E_{j+k-1} - kE_{k-1}E_j - jE_{j-1}E_k \\
&= kV_{j(k-1)} + jV_{(j-1)k} \qquad \text{(a.4)}
\end{aligned}$$

para $j = 0, 1, 2$ y $k = 0, 1, 2$.

(iii) *Covariance between h^k and the regressors z*

Again, the inclusion of h and h^2 together with the rest of the covariates z results in this block of the matrix V. First, note that if $k = 0$, the vector M_{z0} is not affected by a location shift in h variable, therefore $\delta(M_{0z})$ is a vector of zeros of dimension Q:

$$\delta(M_{0z}) = 0_{1\times Q} \quad \text{(a.5)}$$

For $k > 1$, each element is analyzed separately. Let z_q be a covariable in z. The element q of the vector M_{kz} is $Cov(h^k, z_q) = E(h^k z_q) - E(h^k)E(z_q)$, therefore:

$$\begin{aligned}
\delta[Cov(h^k, z_q)] &= \delta[E(h^k z_q) - E(h^k)E(z_q)] \\
&= \delta[E(h^k z_q)] - \delta[E(h^k)]E(z_q) \\
&= kE(h^{k-1} z_q) - kE(h^{k-1})E(z_q) \\
&= kCov(h^{k-1}, z_q)
\end{aligned}$$

for $k = 1, 2$ and $q = 1, 2, \ldots, Q$, where result (a.3) has been used together with the fact that $E(z_q)$ does not change due to a translation of h. Moreover,

$$\delta[E(h^k z_q)] = E\left[\lim_{\varepsilon \to 0} \frac{(h+\varepsilon)^k - h^k}{\varepsilon} \cdot z_q\right] = kE(h^{k-1} z_q)$$

Then, the vector M_{kz} changes as follows:

$$\delta[M_{kz}] = kM_{(k-1)z} \qquad \text{(a.6)}$$

for $k = 1, 2$. Note that when $k = 1$, the change in M_{1z} is a null vector of dimension Q, because M_{0z} is a vector of zeros.

(iv) *Variances and covariances of z*

These moments do not depend on the h distribution, therefore $\delta(M_{zz})$ is a null matrix of dimension $Q \times Q$:

$$\delta[M_{zz}] = 0_{Q \times Q} \qquad \text{(a.7)}$$

Substituting the results (a.4) - (a.7) in $\delta(V)$ gives as a result the second part of equation (3.6).